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The Transverse Systems of Coordinates for the Arctic

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The application of a cartographic grid network of coordinates for the solution of certain geophysical problems associated with the plotting of fields for meteorological and hydrological magnitudes in Polar areas is discussed, and the method is illustrated with corresponding charts and tables of the transformed coordinate system.

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The Cross System of Coordinates for the Arctic

The solution of certain geophysical problems is associated with the plotting of fields for meteorological and hydrological magnitudes and with the computation of their derivatives. In the lower and temperate latitudes the operations can be readily carried out by using the trapeziums of grid network known as squares whose planes and area can be easily determined by any of the degree intervals shown in cartographic tables. Regrettably, the regular grid network does not serve the purpose for polar areas. Because of the pronounced convergence of meridians, it is not possible to find even approximately equidimentional trapezoids in the regular graduated grid. In order to avoid difficulties, some authors divide the surface of the Arctic Ocean as presented on stereographic projection maps by straight lines into squares, the increase resulting at various latitudes with respect to the area near the North Pole can be computed by formulas relating the change of scale with latitude, φ, in comparison to the scale in the area near North Pole: for the straight lines $\mu = \sec^2(45^\circ - \frac{\varphi}{2})$

but for the squares $p = \sec^4 (45^\circ - \frac{\varphi}{2})$.

Thus, for instance, if in the area of the North Pole the side of a square is 100 KM long, in other latitudes the side and area of the square would assume the following values:

TABLE 1

Latitude in degrees	90	80	70	60	50
The length of square side in KM	100	100, 763	103. 110	107. 180	113. 245
The area of square in KM^2	10,000	10, 153. 2	10, 631. 7	11, 487. 6	12, 823. 7

It follows from Table I that within parallel 80°, or even 70°, the numerical differentiation of field elements in the system of plane squares can be regarded as admissible, but beyond the confines of parallel 70 the neglect of the earth's curvature leads to considerable distortions. Therefore, when beyond the confines of latitude 70°, which becomes necessary for instance when /347 analyzing one or another group of fields in the Arctic Ocean and its marginal seas, the system of squares in a plane, tangenital to the point of the North Pole (or secant along any parallel) cannot be accepted.

It appears that a seldom used, but well known in cartography, transverse system of coordinates well serves the objectives indicated above. The system is plotted on the basis of an imagined turn of the globe by 90°, within the usual graduated network, around an axis that is perpendicular to its rotational axis; as a result of this, the earth's poles will be displaced to the (new) equator, but the equator will lie along a pair of meridians 180° distant from each other; passing through both of the earth's (new) poles. It is obvious that the transformation from the usual system of coordinates to the transverse system is admissible only in those cases, to which our problem belongs, where the compression of the earth's globe can be ignored. In order to find the respective formula for the transformation from the normal system of coordinates to the transverse system, let us examine the problem in the general form as it is solved in cartography. Assume that the globe has been arbitrarily displaced in the graduated network so that its North Pole has been displaced from point P to point P' in a latitude φ_0 and longitude λ_0 (Fig. 1). Assume the arbitrary point, M, on the surface of the Northern Hemisphere would normally lie in

latitude φ and longitude λ. Now we have to determine the coordinates for point M in the new, i.e., in the transverse, system of coordinates. For the sake of simplicity, let us count longitude as it is accepted: from 0 to 360°, from the west to the east.

FIGURE 1. Transformation into the new system of coordinates.

As can be seen from Figure 1, in the new coordinate system, the azimuth of point M is a, its zenith distance Z which is connected with the new coordinates in this manner:

$$\varphi' = 90^\circ - Z \quad \text{and} \quad \lambda' = 90^\circ - a. \tag{1}$$

Let us examine the spherical triangle, PP'M formed by the meridians (in the normal system) of points P' and M and by the zenith distance Z. Applying the formulas of spherical trigonometry to this triangle--namely, the formula concerning sine product and the cosine of the adjacent angle, as well as the theorem of sines, we obtain:

$$\left.\begin{aligned} \cos z &= \sin\varphi \cdot \sin\varphi_0 + \cos\varphi \cdot \cos\varphi_0 \cdot \cos(\lambda-\lambda_0), \\ \sin z \cos a &= \sin\varphi \cdot \cos\varphi_0 - \cos\varphi \cdot \sin\varphi_0 \cdot \cos(\lambda-\lambda_0), \\ \sin z \cdot \sin\alpha &= \cos\varphi \cdot \sin(\lambda-\lambda_0), \end{aligned}\right\} \tag{2}$$

In order to obtain a more convenient grid for the polar region, let us place /348
the North Pole of the transverse system of coordinates at the point where the equator is crossed by meridian 270°. Introducing the coordinates of the North Pole into formulas (2) and dividing the third equation by the second, we obtain

$$\left.\begin{aligned} \cos Z &= -\cos\varphi \cdot \sin\lambda \\ \operatorname{tg} a &= \operatorname{ctg}\varphi \cdot \cos\lambda \end{aligned}\right\} \tag{3}$$

from which, recalling formula (1), it follows that:

$$\left.\begin{aligned} \sin\varphi' &= -\cos\varphi \sin\lambda \\ \operatorname{tg}\lambda' &= \operatorname{tg}\varphi \sec\lambda \end{aligned}\right\} \tag{4}$$

if the datum meridian of the transverse system is represented by the arc of the equator extending from $\lambda = 270^{\circ}$ eastward to $\lambda = 90^{\circ}$.

On the basis of formulas (3) and (4) it is possible to determine from the geographical coordinates of a point its latitude and longitude in the transverse system of coordinates.

In cartography, however, the pole of the transverse system of coordinates is usually placed at point $\varphi_0 = 0$, $\lambda_0 = 0$. Introducing the pole coordinates into (2), one obtains not (3), but

$$\left.\begin{aligned} \cos z &= \cos\varphi \, \cos\lambda \\ \operatorname{tg} a &= \operatorname{ctg}\varphi \, \sin\lambda \end{aligned}\right\} \qquad (5)$$

On the basis of formulas (5) special tables have been computed: They are found in the cartographic tables of the Hydrographic Administration VMS (1949) and in the book of Prof. M. D. Solovev (1946), for intervals of 5° of both arguments, but (for intervals of 1°) they are found in the Transactions of the Central Research Institute for aerial photography and Cartography (1945).

The plotting of the graduated grid of the transverse system of coordinates on the map in stereographic projection usually utilized in meterology and oceanology, appears to be very simple. If the center of map (North Pole) is regarded to be the beginning of coordinates and if the axis X is directed along the geographic meridian 180° (in the transverse system, of course, along the equator eastward), but axis Y along the geographic meridian 270° (to the north in the transverse system), the meridians of the transverse system of coordinates

will form semicircles of radius R $(1+\sin\varphi_c)\cdot$ cosec λ', whose center is located at a point of coordinates $X=-R(1+\sin\varphi_c)\,\mathrm{ctg}\,\lambda'$ and $Y=0$. Here and in analogous formulas below, φ_c designates the latitude of the main scale of map on which the graduated grid is plotted. In case the tangent to the pole, and not a secant plane, appears to be the pictorial plane, $1+\sin\varphi_c=2$. The minus mark of the coordinate of center of circle on X axis designates that the meridians intersecting the positive half of axis X appear to be the arcs of circles whose centers lie on the negative half of axis X, and vice versa.

The arcs of circles having radius R $(1+\sin\varphi_c)\,\mathrm{ctg}\,\varphi'$, whose centers lie at points of coordinates $X=0$ and $Y=R(1+\sin\varphi_c)$ cosec λ', appear to be the parallels in a graduated grid transverse system. In the formulas, R is the radius of earth's globe in the main scale of chart, but $R(1+\sin\varphi_c)$ is the distance in millimeters on map from the pole to equator, also in the main scale of map.

Figure 2 shows the map of the Arctic Ocean in the transverse system of coordinates with parallels and meridians drawn at each 5°. They form a /349 regular system of trapeziums not much different from squares whose areas and the length of sides are found in cartographic tables (1). The geographic coordinates of the centers of squares and apexes of their angles can be easily determined on the basis of equations (4), solving them with respect to φ and λ. The computations are as follows: $\cos\varphi=\sqrt{\frac{1+\sin^2\varphi'\cdot\mathrm{tg}^2\lambda'}{1+\mathrm{tg}^2\lambda'}}$

$$\sin\lambda=\frac{\sin\varphi'}{\cos\varphi}. \qquad (6)$$

FIGURE 2. The Map of the Arctic Ocean in the Transverse System of Coordinates

On the basis of formula (6) tables 2 and 3, having angular intervals of 2.5° for both of the arguments, have been computed. The following tables give the geographical latitudes and longitudes of the centers and apexes of the angles of the trapeziums (squares) of the transverse system of coordinates which belong to the second quadrant of a trigonometric circle at whose center lies the North Pole. The Roman numbers in Figure 2 indicate the numbers of the quadrants. /350

TABLE 2

The geographical latitude, φ, of a point with the coordinates of the transverse system listed below.

Longitude

Lat. 30°	60°	62°.5	65°	67°.5	70°	72°.5	75°	77°.5	80°	82°.5	85°	87°.5	90°
30°	48°36'	50°11	54°43'	53°09'	54°28'	55°41'	56°46'	57°44'	58°32'	59°10	59°38'	59°54'	60°00'
27°.5	50°11'	54°53'	53°30'	55°02'	56°28'	57°47'	58°58'	60°00'	60°52	61°34	62°05'	62°24'	62°30'
25°	54°43'	53°30'	55°14'	56°52'	58°23'	59°49'	61°06'	62°14'	63°12'	63°58'	64°32'	64°53'	65°00'
22°.5	53°09'	55°02'	56°52'	58°36'	60°15'	61°47'	63°11'	64°25'	65°29'	66°21'	66°59'	67°22'	67°30'
20°	54°28'	50°28	58°23	60°15'	62°00'	63°40	65°11'	66°33'	67°44'	68°42'	69°25'	69°54'	70°00'
17°.5	55°41'	57°47'	59°49	61°47'	63°40'	65°27'	67°06'	68°37'	69°55'	71°01'	71°49'	72°20'	72°30'
15°	56°46'	58°58'	61°06'	63°11'	65°11'	67°06'	68°55'	70°34'	72°02'	73°16'	74°14'	74°48'	75°00'
12°.5	57°44'	60°00'	62°14'	64°25'	66°33'	68°37'	70°34'	72°24'	74°03'	75°27'	76°33'	77°15'	77°30'
10°	58°32'	60°52'	63°12'	65°29'	67°44'	69°55'	72°02'	74°03'	75°54'	77°31'	78°50'	79°42'	80°00'
7°.5	59°10'	61°34'	63°58'	66°21'	68°42'	71°00'	73°16'	75°27'	77°31'	79°24'	81°00'	82°06	82°30'
5°	59°38'	62°05	64°32'	66°59	69°25	71°49'	74°12'	76°33'	78°50	81°00'	82°56'	84°24'	85°00'
2°.5	59°54'	62°24'	64°53'	67°22	69°51'	72°20	74°48'	77°15'	79°42'	82°06'	84°25'	86°28'	87°30'
0°	60°00'	62°30'	65°00	67°30	70°00	72°30	75°00	77°30'	80°00	82°30	85°00'	87°30'	90°00'

TABLE 3

The geographical longitude ( ) of a point with the coordinates of the transverse system listed below.

Lat.	60°	62.5°	65°	67.5°	70°	72.5°	75° Long.	77.5°	80°	82.5°	85°	87.5°	90°
30°	310°54'	308°38'	306°12'	303°32'	300°39'	297°31'	294°09'	290°36'	286.44'	282°42'	276°35'	274°19'	270°00'
27°.5	313°51'	311°34'	309°04'	306°19'	303°18'	300°01'	296°26'	292°35'	288°27'	284°09'	270°30'	274°47'	270°00'
25°	317°00'	314°43'	312°11'	300°22'	306°16'	302°49'	299°02'	294°54'	200°26'	285°38'	280°37'	275°20'	270°00'
22°.5	320°22'	318°06'	315°34'	312°44'	309°33'	305°59'	302°00'	297°35'	202°45'	287°29'	281°53'	276°00'	270°00'
20°	323°57'	324°45'	319°16'	316°26'	313°13'	309°34'	305°25'	300°20'	297°16'	289°44'	283°29'	276°50'	270°00'
17°.5	327°46'	325°40'	323°16'	320°31'	317°20'	313°38'	309°23'	304°28'	298°51'	292°29'	285°27'	277°53'	270°00'
15°	331°49'	329°52'	327°37'	325°00'	321°55'	318°18'	314°00'	308°56'	302°57'	295°58'	288°01'	279°15'	270°00'
12°.5	336°05'	334°21'	332°19'	329°55'	327°03'	323°36'	319°25'	314°19'	308°04'	300°29'	291°28'	281°09'	270°00'
10°	340°34'	339°06'	337°22'	335°16'	332°44'	329°37'	325°44'	320°50'	314°34'	306°30'	296°18'	283°53'	270°00'
7°.5	345°15'	344°05'	342°42'	341°01'	338°57'	336°21'	333°02'	328°41'	322°50'	314°45'	303°03'	288°20'	270°00'
5°	350°04'	349°16'	348°18'	347°07'	345°39'	343°47'	341°19'	338°00'	333°16'	326°10'	314°53'	296°31'	270°00'
2°.5	355°01'	354°35'	354°06'	353°29'	352°43'	351°44'	350°25'	348°36'	345°53'	341°30'	333°24'	314°57'	270°00'
0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	

TABLE 4

The difference in azmuth at a point with coordinates of the transverse system listed below.

Lat.	Latitude												
	60°	62°.5	65°	67°.5	70°	72°.5	75°	77°.5	80°	82°.5	85°	87°.5	90°
30°	130°54'	133°51	137°00'	140°22'	143°57'	147°46	151°49'	156°05'	160°35	165°15'	170°04'	175°01'	180°00'
27°.5	128°39'	131°34'	134°43'	138°06'	141°35'	145°40	149°52'	154°21'	159°06'	164°05'	169°16'	174°26'	180°00'
25°	126°12'	129°04'	132°11'	135°35'	139°15'	143°17'	147°38'	152°19'	157°21'	162°42'	168°18'	174°06'	180°00'
22°.5	123°32'	126°09'	129°22'	132°44	136°26'	140°31	145°01'	149°55'	155°16'	161°01'	167°07'	173°29'	180°00'
20°	120°39'	123°18'	126°16'	129°33'	133°13'	137°20	141°55	147°03'	152°41'	158°57'	165°39'	172°44'	180°00'
17°.5	117°31'	120°01'	122°49'	125°59'	129°34'	133°39'	138°18'	143°36'	149°37'	156°21'	163°47'	171°44'	180°00'
15°	114°09'	116°26'	119°02'	122°00'	125°25'	129°23'	134°01'	139°25'	145°44'	153°02'	161°19'	170°25'	180°00'
12°.5	110°33'	112°35'	114°54'	117°35'	120°44'	124°22'	128°56'	134°19'	140°50'	148°41'	157°59'	163°36'	180°00'
10°	106°44'	108°27'	110°25'	112°45'	115°30'	118°50'	112°57'	128°04	134°36	142°50'	153°15'	165°53'	180°00'
7°.5	102°44'	104°05'	105°38	107°30	109°44'	112°29'	115°58'	120°20'	126°31'	134°45'	146°10'	161°31'	180°00'
5°	98°35'	99°30'	100°35'	101°53'	103°28'	105°27'	108°01'	111°23'	116°18	123°31'	134°54'	153°24'	180°00'
2°.5	94°19'	94°47	95°24'	96°01'	96°50'	97°52'	99°14'	101°07'	103°53'	108°19	116°29'	134°57'	180°00'
0°	90°	90°	90°	90°	90°	90°	90°	90°	90°	90°	90°	90°	90°

On the basis of the spherical triangle, PP'M shown in Figure 1, it is also easy to find the correlation between the azimuth at Point M in a geographic system of coordinates a and in a transverse system a'. It is expressed by the following formula:

$$\sin(a'-a)=\frac{\cos \lambda'}{\cos \varphi'} \qquad (7)$$

By using formula (7) the magnitude $a'-a=\beta$ (Table 4) has been computed for the second quadrant.

In order to determine the corresponding values for the fourth and first quadrants from Tables 2, 3, and 4, values for $180^{\circ}-\lambda'$ should be entered into its columns, but for the third and fourth quadrants, the value of minus φ' should be entered into its lines.

On the basis of trigonometric correlations of the coordinates and azimuths, the points for the third, fourth, and the first quadrants in connection with the magnitudes for the second quadrant are determined by the formulas listed in Table 5.

TABLE 5

Quarters	II	III	IV	I
Latitude	φ_{II}	$\varphi_{III}=\varphi_{II}$	$\varphi_{IV}=\varphi_{II}$	$\varphi_{I}=\varphi_{II}$
Longitude	λ_{II}	$\lambda_{III}=360^{\circ}-\lambda_{II}$	$\lambda_{IV}=\lambda_{II}-180^{\circ}$	$\lambda_{I}=540^{\circ}-\lambda_{II}$
Az.	$\alpha=\alpha'-\beta$	$\alpha=\alpha'+\beta-180^{\circ}$	$\alpha=\alpha'-\beta+180^{\circ}$	$\alpha=\alpha'+\beta$

In conclusion, I express my gratitude to the younger scientific collaborator, Iu. A. Shishkov, for the computation of the tables and the laboratory research man, V. D. Burmistrova, for plotting the graduated grid in the transverse system of coordinates.

FIGURE 1.

Transformation into the new system of coordinates.

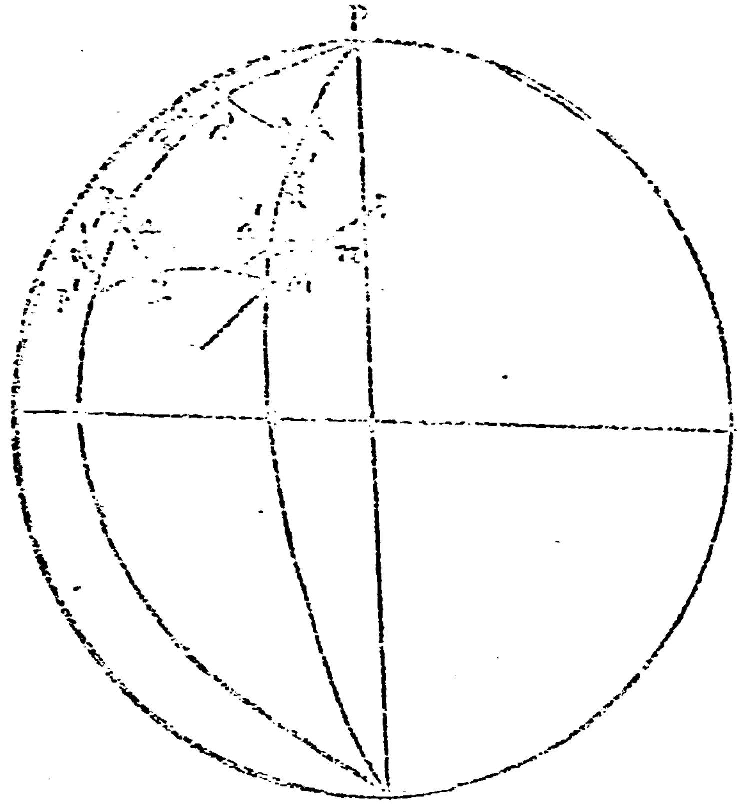
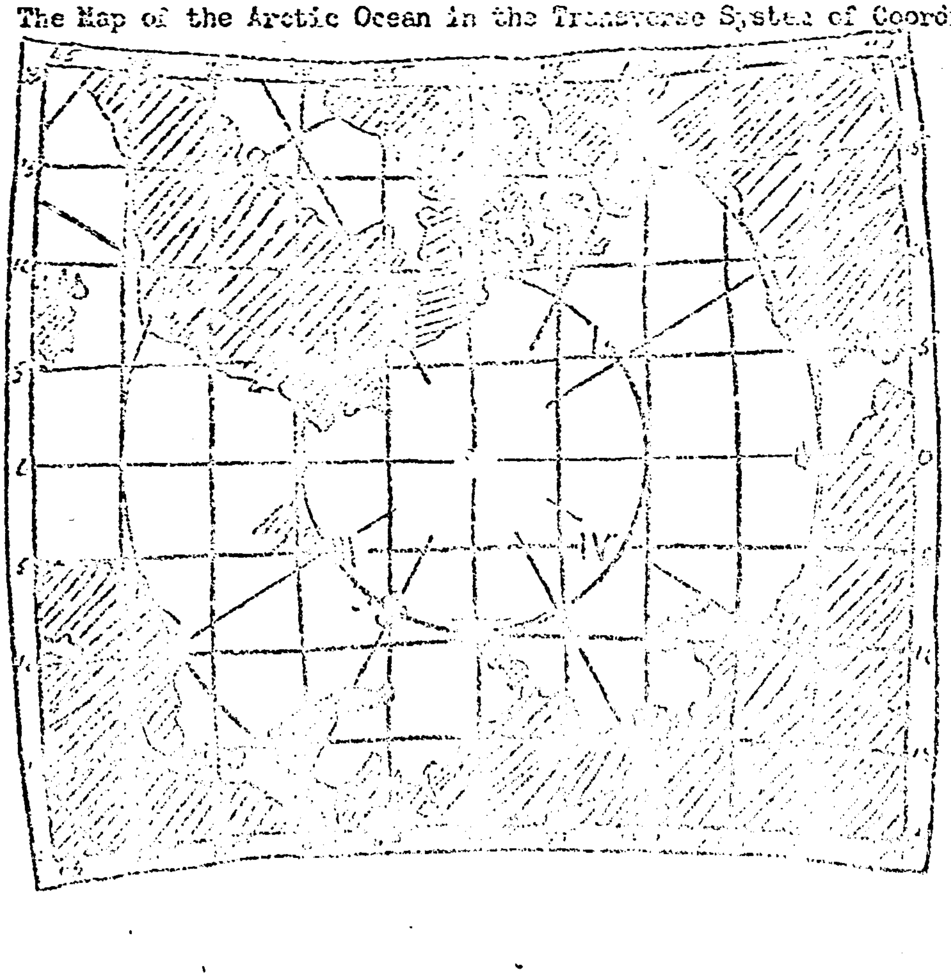


FIGURE 2 The Map of the Arctic Ocean in the Transverse System of Coordianates



BIBLIOGRAPHY

ANON. (The cartographic tables of Hydrographic Administration VMS). 1949

ANON. (The cartographic tables) Trudi Instituta geodezii, aerofotos'emki i [illegible] kartografii, V. 41, 1945

SOLOV'EV, M. D. (Cartographic projections) Geodezizdat, M. 1946

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